Funkcia jednej reálnej premennej

Derivácia funkcie

1. Vypočítajte deriváciu funkcie

1. $y = x^4 - 7x^3 + 2x + 3$

$$[4x^3 - 21x^2 + 2]$$

2. $y = (x^3 - 2x + 1)(x^4 - 5x^2 + 10)$

$$[7x^6 - 35x^4 + 4x^3 + 60x^2 - 10x - 20]$$

3. $y = 2x + \sqrt[4]{x} + \sqrt[6]{x} + \sqrt[9]{x}$

$$[2 + \tfrac{1}{4}x^{\frac{-3}{4}} + \tfrac{1}{6}x^{\frac{-5}{6}} + \tfrac{1}{9}x^{\frac{-8}{9}}]$$

4. $y = \sqrt[4]{x^2\sqrt{x^4\sqrt{x^3}}}$

$$[\tfrac{19}{16}x^{\frac{3}{16}}]$$

5. $y = x^{18} + 4x^3\sqrt{x} + 4\sqrt[3]{x^2} - \dfrac{3}{x^5} + \dfrac{5}{\sqrt[3]{x^2}}$

$$[18x^{17} + 14x^{\frac{5}{2}} + \tfrac{8}{3}x^{-\frac{1}{3}} + 15x^{-6} - \tfrac{10}{3}x^{-\frac{5}{3}}]$$

6. $y = \dfrac{3x}{1 + x^3}$

$$\left[\frac{3 - 6x^3}{(1 + x^3)^2}\right]$$

7. $y = (4 + 11x)^{15}$

$$[165(4 + 11x)^{14}]$$

8. $y = \sqrt[3]{(x - 3)^5}$

$$\left[\frac{5}{3}\sqrt[3]{(x - 3)^2}\right]$$

9. $y = \cos(3x+8)$

$$[-3\sin(3x+8)]$$

10. $y = (2-x^2)\sin x + (3+x^3)\cos x$

$$[2(1+x^2)\cos x - (x^3+2x+3)\sin x]$$

11. $y = tg\sqrt[5]{5+x^5}$

$$\left[\frac{x}{\sqrt[5]{(5+x^5)^4}\cos^2\sqrt[5]{5+x^5}}\right]$$

12. $y = 2^{x^2}$

$$\left[2^{x^2+1}x\ln 2\right]$$

13. $y = \ln(x+\sqrt{1+x^2})$

$$\left[\frac{1}{\sqrt{1+x^2}}\right]$$

14. $y = arctg(tgx)$

$$[1]$$

15. $y = \sin\dfrac{1+3x+2x^2}{1-x^2} + 3^{\frac{x^2+x+1}{1-x^2}}$

$$\left[\frac{3(x+1)^2}{(1-x^2)^2}\cos\frac{1+3x+2x^2}{1-x^2} + \frac{(x^2+4x+1)\ln 3}{(1-x^2)^2}3^{\frac{x^2+x+1}{1-x^2}}\right]$$

16. $y = cotg(5x^2)$

$$\left[-\frac{10x}{\sin^2(5x^2)}\right]$$

17. $y = \cos(7^{\frac{1}{2x}})$

$$\left[\frac{\sin(7^{\frac{1}{2x}})7^{\frac{1}{2x}}\ln 7}{2x^2}\right]$$

18. $y = \log \frac{2^x+1}{x^2+1}$

$$\left[\frac{2^x \ln 2(x^2+1) - 2x(2^x+1)}{(2^x+1)(x^2+1)\ln 10}\right]$$

19. $y = 5^{\frac{x^2}{3^x}}$

$$\left[\frac{5^{\frac{x^2}{3^x}} \ln 5(2x - \ln 3)}{3^x}\right]$$

20. $y = \dfrac{x}{x^2+2} - \dfrac{2}{\sqrt[4]{(x+3)^5}}$

$$\left[\frac{2-x^2}{(x^2+2)^2} - \frac{5}{2\sqrt[4]{(x+3)^9}}\right]$$

21. $y = (\sqrt{x} + \sin 2x)\left(\log_3 x + \dfrac{1}{x^3}\right)$

$$\left[(\frac{1}{2\sqrt{x}} + 2\cos(2x))(\log_3 x + x^{-3}) + (\sqrt{x} + \sin(2x))(\frac{1}{x\ln 3} - 3x^{-4})\right]$$

22. $y = \sqrt{4x - x^2} + 4\arcsin\frac{\sqrt{x}}{2}$

$$\left[\frac{4-x}{\sqrt{x(4-x)}}\right]$$

23. $y = \sqrt[3]{sin(2x-5)} + \dfrac{e^{2x+3}+5}{x^2}$

$$\left[\frac{2\cos(2x-5)}{3\sin^2(2x-5)} + \frac{2e^{2x+3}(x-1)-10}{x^3}\right]$$

24. $y = \sqrt[5]{\dfrac{1+x}{1+2x}} + x\sqrt{4x^2+1}$

$$\left[-\frac{1}{5(1+2x)^2}\sqrt[5]{(\frac{1+2x}{1+x})^4} + \frac{8x^2+1}{\sqrt{4x^2+1}}\right]$$

25. $y = tg^3 x + tg x^3 + tg 3x + 3tg x + tg\frac{3}{x} + tg\frac{x}{3}$

$$\left[\frac{3tg^2x}{\cos^2 x}+\frac{3x^2}{\cos^2 x^3}+\frac{3}{\cos^2(3x)}+\frac{3}{\cos^2 x}-\frac{3}{x^2\cos^2\frac{3}{x}}+\frac{1}{\cos^2\frac{x}{3}}\right]$$

26. $y=\dfrac{2}{\ln(1+\sqrt{x})}$

$$\left[-\frac{1}{[\ln^2(1+\sqrt{x})(\sqrt{x}+x)]}\right]$$

27. $y=\log_3(\log_2(\ln x))$

$$\left[\frac{1}{x\ln 2\ln 3\ln x\log_2\ln x}\right]$$

28. $y=\ln\frac{x^2+1}{2x^3-1}$

$$\left[\frac{-2x(x^3+3x-1)}{(x^2+1)(2x^3-1)}\right]$$

29. $y=\cos x\sin(\cos\frac{1}{x})$

$$\left[\frac{\cos x\cos\cos\frac{1}{x}\sin\frac{1}{x}}{x^2}-\sin x\sin\cos\frac{1}{x}\right]$$

30. $y=x\arccos x-\dfrac{x}{\sqrt{1-x^2}}$

$$\left[\arccos x+\frac{x^3-x-1}{(1-x^2)\sqrt{1-x^2}}\right]$$

31. $y=\ln\frac{1+\sqrt{1+x^2}}{x}-\sqrt{x^2+1}$

$$\left[\frac{1-x^2}{x\sqrt{1+x^2}}\right]$$

32. $y=(5x-2)arctg(\frac{4}{\sqrt{x}})$

$$\left[5arctg\tfrac{4}{\sqrt{x}}-\tfrac{2(5x-2)}{\sqrt{x}(x+16)}\right]$$

33. $y=\dfrac{2}{3}arccotg(\frac{x}{1-x^2})$

$$\left[-\frac{2(1+x^2)}{3(1-x^2+x^4)}\right]$$

34. $y = \arcsin(\sqrt{\frac{3-x}{3+x}}) + x^2 tgx$

$$\left[2x\,\mathrm{tg}\,x + \frac{x^2}{\cos^2 x} - \frac{\sqrt{3}}{\sqrt{2(3-x)}(3+x)}\right]$$

35. $y = 10^{\sin^3(2+x^4)}$

$$\left[12x^3 10^{\sin^3(2+x^4)} \ln 10 \sin^2(2+x^4)\cos(2+x^4)\right]$$

36. $y = e^{x \ln(\cos 2x)}$

$$\left[e^{x\ln(\cos 2x)}\left[\ln\cos(2x) - 2xtg(2x)\right]\right]$$

37. $y = 2^{\frac{x}{\sqrt{\sin x}}}$

$$\left[2^{\frac{x}{\sqrt{\sin x}}} \ln 2 \frac{2\sin x - x\cos x}{2\sqrt{\sin^3 x}}\right]$$

38. $y = cotg(e^{x^2+3x-2})$

$$\left[-\frac{(2x+3)e^{x^2+3x-2}}{\sin^2 e^{x^2+3x-2}}\right]$$

39. $y = (x^2+3)^{\sin x}$

$$\left[(x^2+3)^{\sin x}\left[\cos x \ln(x^2+3) + \frac{2x\sin x}{x^2+3}\right]\right]$$

40. $y = \sqrt{1 - \frac{\sqrt{x}}{1+x}} - (\frac{2+5x}{\ln x})^4$

41. $y = \ln^4(x + \sqrt{\frac{1-3x^3}{\sin 5x}})\,\mathrm{arctg}\,\frac{2+x}{2-x}$

42. $y = \dfrac{\cos\ln\sqrt{e^{4x}+4}}{13}$

43. $y = \log_3 \log \sqrt[5]{\frac{1+x}{\sin x}} - \mathrm{tg}\,\frac{x^2-4}{x}$

44. $y = \dfrac{\ln(1+\mathrm{tg}^2 x)}{2\,\mathrm{cotg}\,x\,\mathrm{arctg}\sin x}$

45. $y = \dfrac{\log_2 3^x - \log_3(x^8\sqrt{x})}{\mathrm{arctg}(\frac{\sqrt{x}+\sqrt{e}}{\sqrt[4]{x}})^e}$

46. $y = \ln \sin \frac{1+3x+10x^3}{1-x^4} + 3^{\frac{x^2+x+1}{1-x^2}}$

47. $y = \cos^6 7^{\frac{1}{2x}} \log^4 \frac{2^x+1}{x^2+1} + (\sin x)^{\operatorname{tg} x}$

48. $y = x^{5^x} - \dfrac{10^{\sin^3 \log(2+x^4)} \operatorname{arctg} \sqrt{x\sqrt{x\sqrt{x}}}}{\operatorname{tg} 5x^4}$

49. $y = \sqrt[5]{\dfrac{\arcsin 3x}{\sqrt[3]{4x+1}}} + \left(\dfrac{x}{x+1}\right)^x$

50. $y = \dfrac{\sqrt[33]{1+x} - \sin \operatorname{tg}^2 \frac{x^2}{2}}{\log_{\frac{1}{7}}(\sqrt{x-3}+1)}$

51. $y = \dfrac{2x^7 + x^4\sqrt{x} + \sin\sqrt{x}}{\operatorname{arctg} \frac{x+1}{x}} - (\cos x)^{\ln x}$

52. $y = \dfrac{x^4 \arcsin x}{\sqrt{1-x^2}} \ln \sqrt{1-x^2} \operatorname{arctg}^2 \sqrt{\sin 3x}$

53. $y = x \ln \frac{1-\sqrt{\sin x}}{1+\cos x} + \sin \frac{e}{\sqrt{e}} + \ln^{41} \log \frac{5^{\sqrt{x}} 3^{\frac{x+1}{2x}}}{\sqrt[5]{x}}$

54. $y = \dfrac{\sqrt[4]{x\sqrt{x\sqrt[3]{x^3x^7}}}}{\cos^{14} \ln 35} + (\operatorname{tg} 3x)^{\operatorname{tg} 2x} + (\sin\sqrt{x})^{\arcsin\sqrt{x}}$

55. $y = 3^{\operatorname{arctg} \frac{x^2}{x+1}} \ln \sin \sqrt{\cos 3x}$

56. $y = \dfrac{\sqrt[3]{(x+1)(3x-9)}}{\arcsin \cos 2x} + \ln \frac{x^2-x}{\sqrt{x}}$

57. $y = \cos^2 4x \operatorname{arctg} \ln(2x+1)^2 + [\sin(x^2+2x)]^x$

58. $y = \operatorname{arctg}(e^{x^2+3}3x^2) + \ln \frac{x^2-9}{x-3} \sin \sqrt[3]{\sqrt{x^6}}$

59. $y = \dfrac{\sqrt{x^2+x-2}}{1-\log\sqrt{x^2}} + \dfrac{3^{x^2+22}}{e^{3x}\ln 3x}$

60. $y = \dfrac{\frac{\sqrt[9]{\sin\cos x^2}}{x^4+3x^2}}{\frac{x^3-27}{x^2+3x+9}}$

2. Logaritmickým derivovaním vypočítajte deriváciu funkcie

1. $y = (e^x)^{\ln x}$

$$\left[(e^x)^{\ln x}(1+\ln x)\right]$$

2. $y = x^{5^x}$

$$\left[x^{5^x} 5^x (\ln 5 \ln x + x^{-1})\right]$$

3. $y = (\sin x)^{e^x}$

$$\left[(\sin x)^{e^x} e^x (\ln \sin x + cotgx)\right]$$